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Reg. No. : ....

Code No. : 5548 **Sub. Code : KMAM43**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Fourth Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :s

1. A point P where ———— is called a point of inflexion.

   (a) $\bar{r} = \bar{o}$ (b) $\bar{r}' = \bar{o}$

   (c) $\bar{r}'' = \bar{o}$ (d) $\bar{r}' . \bar{r}'' = o$

2. $[\bar{r}', \bar{r}'', \bar{r}''']$ is

   (a) $K^2 \tau$ (b) $K \tau^2$

   (c) $K^2 (u^1)^{-6}$ (d) O

3. A helix of constant curvature is necessarily

(a) a sphere (b) a spherical helix

(c) a cylindrical helix (d) a circular helix

4. A necessary and sufficient condition that a curve be a straight line is

(a) $K = 0$ at all points (b) $\tau = 0$ at all points

(c) $[\overline{r}', \overline{r}'', \overline{r}'''] = 0$ (d) $[\dot{\overline{r}}, \ddot{\overline{r}}, \dddot{\overline{r}}] = 0$

5. Eliminating u and v from $x = u \cos hv$, $y = u \sin hv$, $z = u^2$, the constraint equation is

(a) $x^2 - y^2 = z$ (b) $x^2 + y^2 = z$

(c) $x^2 + y^2 = -z$ (d) $x^2 - y^2 = -z$

6. The formula for ds^2 is

(a) $Edu^2 + Fdudv + Gdv^2$

(b) $E\,du^2 + 2Fdudv + Gdv^2$

(c) $E\,du + 2Fdudv + Gdv$

(d) $E\,dv^2 + 2Fdudv + Gdu^2$

7. If the parametric curves are orthogonal then the curve $v = c$ will be geodesic if and only if

(a) E is a function of v only

(b) E is a function of u only

(c) G is a function of v only

(d) G is a function of u only

8. Orthogonal trajectories are called

(a) orthogonal parallels

(b) orthogonal geodesic

(c) geodesic parallels

(d) geodesic trajectories

9. The second fundamental form is

(a) $Ldu^2 + 2Mdudv + Ndv^2$

(b) $Ldu^2 + Mdudv + Ndv^2$

(c) $Ldu + 2Mdu^2dv^2 + Ndv$

(d) $Ldu^2 - 2Mdudv + Ndv^2$

10. The Gaussian curvature K is defined by

(a) $\dfrac{LN + M^2}{EG - F^2}$ (b) $\dfrac{LN - M^2}{EG - F^2}$

(c) $\dfrac{LN + M^2}{EG + F^2}$ (d) $\dfrac{EG - F^2}{LN - M^2}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Calculate the curvature and torsion of the cubic curve given by $r = \left(u, u^2, u^3\right)$.

Or

(b) Prove that $\left[\bar{r}', \bar{r}'', \bar{r}'''\right] = K^2 \tau$.

12. (a) Show that the osculating plane at P has, in general, three point contact with the curve at P.

Or

(b) Show that the involutes of a circular helix are plane curves.

13. (a) Show that a proper parametric transmission either leaves every normal unchanged or reverses every normal.

Or

(b) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

14. (a) In the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the section by the planes $z =$constant.

Or

(b) Prove that the curves of the family $v^3 / u^2 =$constant are geodesics on a surface with metric $v^2du^2 - 2uv\ dudv + 2u^2dv^2$ (where $u > 0,\ V > 0$).

15. (a) Find the geodesic curvature of the parametric curve $v = c$.

Or

(b) Prove the Euler's formula

$$K = K_a \cos^2 \psi + K_b \sin^2 \psi .$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Obtain the Serret-Frenet formulae.

Or

(b) Find the curvature and the torsion of the curve $r = \{a(3u - u^3), 3au^2, a(3u + u^3)\}$.

17. (a) Show that the spherical indication of a curve is a circle if and only if the curve is a helix.

Or

(b) Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in a constant ratio.

18. (a) Show that the parametric curves on the sphere given by $x = a \sin x \cos v$, $y = a \sin u \sin v$, $z = a \cos u$, $0 < u < \pi / 2$, $0 < v < 2\pi$, form on orthogonal system. Determine the two families of curves whch meet the curves $v =$ constant at angles of $\pi / 4$ and $3\pi / 4$.

Or

(b) Show that on a right helicoids, the family of curves orthogonal to the curves $u \cos v = \text{constant}$ in the family $\left(u^2 + a^2\right)$ $\sin^2 v =$ constant.

19. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$, for all values of u.

Or

(b) Find the geodesics on a surface of revolution.

20. (a) Define lines of curvature and characterize the lines of curvature.

Or

(b) State and prove Hilbert's lemma.